$$=\frac{1}{\Gamma(\frac{n}{2})}\int_0^{\frac{t}{2}} u^{\frac{n}{2}-1}e^{-u}du$$

$$=\frac{1}{\Gamma(\frac{n}{2})}\int_0^{\frac{t}{2}} x^{\frac{n}{2}-1}e^{-x}dx$$

ここで，$\Gamma(\frac{n}{2})=\int_0^{\infty} x^{\frac{n}{2}-1}e^{-x}dx$

$$\therefore \chi_n^2(t)=\frac{\int_0^{\frac{t}{2}} x^{\frac{n}{2}-1}e^{-x}dx}{\int_0^{\infty} x^{\frac{n}{2}-1}e^{-x}dx} \quad \cdots\cdots ②$$

(1) nが偶数の場合　②の

$$分子=\int_0^{\frac{t}{2}} x^{\frac{n}{2}-1}e^{-x}dx$$

$$=\left[-e^{-x}x^{\frac{n}{2}-1}\right]_0^{\frac{t}{2}}+\int_0^{\frac{t}{2}} e^{-x}(\frac{n}{2}-1)x^{\frac{n}{2}-2}dx$$

$$=-e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-1}+(\frac{n}{2}-1)\int_0^{\frac{t}{2}} e^{-x}x^{\frac{n}{2}-2}dx$$

以下，部分積分法をくり返して

$$=-e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-1}-(\frac{n}{2}-1)e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-2}-(\frac{n}{2}-1)(\frac{n}{2}-2)e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-3}$$

$$\cdots\cdots\cdots-(\frac{n}{2}-1)!\,e^{-\frac{t}{2}}+(\frac{n}{2}-1)!$$

$$分母=\Gamma(\frac{n}{2})=(\frac{n}{2}-1)!$$

$$\therefore \chi_n^2(t)=1-e^{-\frac{t}{2}}\{1+\frac{1}{1!}(\frac{t}{2})+\frac{1}{2!}(\frac{t}{2})^2+\cdots\cdots+\frac{1}{(\frac{n}{2}-1)!}(\frac{t}{2})^{\frac{n}{2}-1}\}$$

ここで，$\chi_n^{2\prime}(t)=1-\chi_n^2(t)$とおくと