$$\chi^{2\prime}(t)=e^{-\frac{t}{2}}\{1+\frac{1}{1!}(\frac{t}{2})+\frac{1}{2!}(\frac{t}{2})^2+\cdots\cdots+\frac{1}{(\frac{n}{2}-1)!}(\frac{t}{2})^{\frac{n}{2}-1}\}\cdots\cdots\cdots\cdots③$$

(2) nが奇数（$n\geqq 3$，$n=1$のときは省略）の場合　②の

$$分子=\int_0^{\frac{t}{2}}x^{\frac{n}{2}-1}e^{-x}\,dx$$

$$=-e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-1}-(\frac{n}{2}-1)\,e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-2}-(\frac{n}{2}-1)(\frac{n}{2}-2)\,e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{n}{2}-3}$$

$$-\cdots\cdots\cdots-(\frac{n}{2}-1)(\frac{n}{2}-2)\cdots\cdots\frac{5}{2}\cdot\frac{3}{2}\,e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{1}{2}}$$

$$+(\frac{n}{2}-1)(\frac{n}{2}-2)\cdots\cdots\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\int_0^{\frac{t}{2}}e^{-x}x^{-\frac{1}{2}}dx$$

$$分母=\int_0^{\infty}x^{\frac{n}{2}-1}e^{-x}dx$$

$$=(\frac{n}{2}-1)(\frac{n}{2}-2)\cdots\cdots\cdots\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma(\frac{1}{2})$$

$$=(\frac{n}{2}-1)(\frac{n}{2}-2)\cdots\cdots\cdots\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}$$

$$\therefore\quad \chi^2_n(t)=\frac{1}{\sqrt{\pi}}\int_0^{\frac{t}{2}}e^{-x}x^{-\frac{1}{2}}dx-\frac{1}{\sqrt{\pi}}e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{1}{2}}\{2+\frac{2\cdot 2}{1\cdot 3}(\frac{t}{2})$$

$$+\frac{2\cdot 2\cdot 2}{1\cdot 3\cdot 5}(\frac{t}{2})^2+\cdots\cdots+\frac{2\cdot 2\cdot 2\cdots\cdots\ 2}{1\cdot 3\cdot 5\cdots\cdots(n-2)}(\frac{t}{2})^{\frac{n-3}{2}}\}$$

$$=\frac{1}{\sqrt{\pi}}\int_0^{\frac{t}{2}}e^{-x}x^{-\frac{1}{2}}dx-\frac{2}{\sqrt{\pi}}e^{-\frac{t}{2}}(\frac{t}{2})^{\frac{1}{2}}\{1+\frac{t}{1\cdot 3}+\frac{t^2}{1\cdot 3\cdot 5}+\cdots\cdots$$

$$+\cdots\cdots+\frac{t^{\frac{n-3}{2}}}{1\cdot 3\cdot 5\cdots\cdots(n-2)}\}$$

ここで，　$I=\dfrac{1}{\sqrt{\pi}}\displaystyle\int_0^{\frac{t}{2}}e^{-x}x^{-\frac{1}{2}}dx$　において，

$x=\dfrac{u^2}{2}$ とおくと　$x^{-\frac{1}{2}}=\dfrac{\sqrt{2}}{u}$

x	$0\ \rightarrow\ \frac{t}{2}$
u	$0\ \rightarrow\ \sqrt{t}$

，　$dx=u\,du$

$$\therefore\quad I=\frac{1}{\sqrt{\pi}}\int_0^{\sqrt{t}}e^{-\frac{u^2}{2}}\ \frac{\sqrt{2}}{u}\cdot u\,du$$

$$=\frac{2}{\sqrt{2\pi}}\int_0^{\sqrt{t}}e^{-\frac{u^2}{2}}du$$

よって，